Your Understanding

Examples 1-4

Write an inequality for each sentence.

(p. 24

- 1. Your speed must be 55 miles per hour or less.
- 2. The game is recommended for ages greater than 6.

Examples 5 and 6

State whether each inequality is true or false for the given value.

(p. 244)

3. x - 11 < 9, x = 20 **4.** $42 \ge 6a, a = 8$

5.
$$\frac{n}{3} + 1 \le 6, n = 15$$

Examples 7 and 8 (p. 244)

Graph each inequality on a number line.

6. n > 4

 $p \leq 2$

8. $x \ge 0$

9. a < 7

Practice and Problem Solving

•

Step-by-Step Solutions begin on page R20.
 Extra Practice is on page EP14.

Examples 1–4

Write an inequality for each sentence.

10. To be a U.S. senator, you must be 30 years of age or older.

11. For a group of 10 or more, an 18% tip is already included.

12. The maximum occupancy must be less than 512 people.

13. The phone costs no more than \$25.

14. You must spend more than \$50 to receive a discount.

15. The heavyweight division is greater than 200 pounds.

Examples 5 and 6 (p. 244)

State whether each inequality is true or false for the given value.

16. 12 + a < 20, a = 9 **17.** 15 - k > 6, k = 8

18. -3y < 21, y = 8

19. $32 \le 2x$, x = 16

20. $\frac{n}{4} \ge 5$, n = 12

21. $\frac{-18}{x} > 9, x = -2$

Examples 7 and 8 (p. 244) Graph each inequality on a number line.

22. x > 6

23. a > 0

24. y < 8

25. *h* < 2

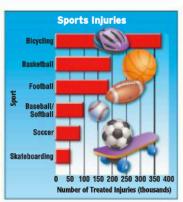
26. $w \le 3$

27. $p \ge 7$

28. $1 \le n$

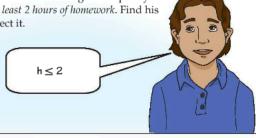
29. $4 \ge d$

- SPORTS The graph shows the number of children ages 5–14 recently treated in U.S. emergency rooms.
 - **a.** In which sport(s) were more than 150,000 children injured?
 - **b.** In which sport(s) were at least 75,000 children injured?
 - c. Of the sports listed, which have fewer than 125,000 injuries?
 - d. Write an inequality comparing the number treated for soccerrelated injuries with those treated for football-related injuries.



H.O.T. Problems

31. FIND THE ERROR Roberto is writing an inequality for the expression at least 2 hours of homework. Find his mistake and correct it.



- **32. CHALLENGE** If x = 3, is the following inequality *true* or *false*? Explain. $\frac{108}{12} + x \ge 15 - 4x + 9$
- **33.** Write MATH If a < b and b < c, what is true about the relationship between a and c? Explain your reasoning and give examples using both positive and negative values for a, b, and c.



MA.8.A.4.2

34. Conner can spend no more than 4 hours at the swimming pool today. Which graph represents the time that Conner can spend at the pool?



35. Which inequality matches the sentence below?

Members must be 18 years of age or older.

G.
$$m \ge 18$$

H.
$$m < 18$$

36. SHORT RESPONSE Graph the inequality x < -8 on a number line.



37. SHOPPING Marisa bought 4 paperback books, each at the same price. The tax on her purchase was \$2.35, and the total was \$34.15. Write and solve an equation to find the price of each book. (Lesson 4-2C)

ALGEBRA Solve each equation. (Lesson 4-2B)

38.
$$9 + 5y = 19$$

39.
$$-6 = 4 + 2x$$

40.
$$8 - k = 17$$

41.
$$2 = 18 - 4d$$

246 Chapter 4 Equations and Inequalities



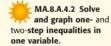
One-Step Inequalities



Main Idea

Solve and graph onestep inequalities by using the Addition or **Subtraction Properties** of Inequality.

NGSSS

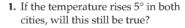






Solve Inequalities by Addition or Subtraction

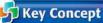
WEATHER The temperature at 7 A.M. in three Florida cities is shown. The temperature in Miami is less than the temperature in Jacksonville.





2. Would it be colder in Tampa or Jacksonville if the temperature in both cities dropped 10°? Explain.

The examples above demonstrate properties of inequalities.



Properties of Inequality

Words

When you add or subtract the same number from each side of an inequality, the inequality remains true.

Symbols

For all numbers, a, b, and c,

1. if
$$a > b$$
, then $a + c > b + c$ and $a - c > b - c$.

2. if
$$a < b$$
, then $a + c < b + c$ and $a - c < b - c$.

Examples

$$2 > -3$$
 $2 + 5 > -3 + 5$

$$3 < 8$$
 $3 - 4 < 8 - 4$

These properties are also true for $a \ge b$ and $a \le b$.

Solving an inequality means finding values that make the inequality true.

ns To inequality, replace the variable with a value that satisfies the solution. In Example 1, replace n with 22.

n - 8 < 15 22 - 8 < 15

14 2 15 ice 14 < 15, the tement is true.

EXAMPLES Solve Inequalities



Solve n - 8 < 15. Graph the solution set on a number line.

$$n - 8 < 15$$

Write the inequality.

Addition Property of Inequality

Simplify.

Graph the solution set.



Draw an open dot at 23 with an arrow to the left.



equal to a, then a is less than or equal to -11.



Solve $-4 \ge a + 7$. Graph the solution set on a number line.

$$-4 \ge a + 7$$

Write the inequality.

Subtraction Property of Inequality

$$-11 \ge a \text{ or } a \le -11$$
 Simplify.

The solution is $a \le -11$.

Draw a closed dot at -11 with an arrow to the left.



CHECK Your Progress

Solve each inequality. Graph the solution set on a number line.

a.
$$t + 3 > 12$$

b.
$$n + \frac{1}{2} \ge 4$$
 c. $y - 1.5 < 2$

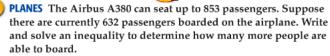
c.
$$y - 1.5 < 2$$



Real-World Link • • • •

The Airbus A380 is one of the largest airplanes in the world.

Real-World EXAMPLE



The phrase up to means less than or equal to. Let p = the number of passengers left to board.

Estimate 850 - 650 = 200

Check for Reasonableness 200 ≈ 221 ✓

At most, 221 people are able to board.



CHECK Your Progress

d. WEATHER An F1 tornado has wind speeds that are at least 73 miles per hour. An F2 tornado has wind speeds that are at least 113 miles per hour. Write and solve an inequality to determine how much the winds of an F1 tornado need to increase to become at least an F2 tornado.



Examples 1 and 2 (pp. 247–248)

Solve each inequality. Graph the solution set on a number line.

- **1.** *b* + 5 > 9
- **2.** $12 + n \le 4$
- 3. x 4 < 10

Example 3 (p. 248) 4. WEATHER Concord, New Hampshire, receives an average of 37 inches of precipitation per year, and there has been 13 inches so far this year. Write and solve an inequality to determine how much more precipitation Concord can get and stay at or below the average.

Practice and Problem Solving

Step-by-Step Solutions begin on page R20.
Extra Practice is on page EP14.

Examples 1 and 2 (pp. 247-248)

Solve each inequality. Graph the solution set on a number line.

- 5) $5 + x \le 18$ 8. 3 < y + 8
- **6.** $10 + n \ge -2$ **9.** c + 10 < 9
- 7. -4 < k + 610. $g - 4 \ge 13$

- **11.** $2 + m \ge 3.5$
- **12.** $q + 0.8 \le -0.5$
- **13.** v 6 > 2.7

- **14.** p 4.8 > -6
- **15.** $d + 1\frac{2}{3} \le \frac{1}{2}$
- **16.** $5 > f + 1\frac{1}{4}$

Join the City

Basketball

Division A (ages 18-23)

Division B (ages 24–29) Division C (ages 30 and up)

League

Example 3 (p. 248)

- BASKETBALL Amos, who is 15 years old, is thinking about joining the City Basketball League. Write and solve an inequality to determine how many years until he is able to join.
- 18. ANIMALS Hippos weigh up to 5,300 pounds. Write and solve an inequality that describes how much weight a young hippo could gain if its current weight is 2,200 pounds.
- **19. MULTIPLE REPRESENTATIONS** The inequality y < x + 1 is a linear inequality.
 - WORDS Use the definition of a linear function to make a conjecture about the definition of a linear inequality.
 - **b. GRAPHS** Graph the function y = x + 1. This is the line that determines the inequality. Similar to the open/closed dot, the line will be dashed or solid. Should the graph of this inequality be a dashed or solid line?
 - c. NUMBERS Choose a point not on the line and substitute the x- and y-values into the inequality. If the resulting inequality is true, shade the area containing the point. If it is false, shade the area that does not contain the point.
- 20. FIND THE DATA Refer to the Data File on pages 2–5. Choose some data and write a real-world problem in which you would need to solve an inequality using addition or subtraction.



H.O.T. Problems

CHALLENGE Determine whether each equation or inequality has no solution, one solution, or more than one solution.

21.
$$y - y = 0$$

22.
$$x + 4 = 9$$

23.
$$x + 4 > 9$$

24.
$$y > y + 1$$

- **25. OPEN ENDED** Write two different inequalities that each have the solution x < 9. One inequality should be solved using addition properties, and the other should be solved using subtraction properties.
- **26.** Write MATH Write a word problem that has the solution $w \le 200$.



MA.6.A.3.2, MA.8.A.4.2

- 27. SHORT RESPONSE Kali has \$80 to go clothes shopping. She spends \$25 on a new shirt. Write an inequality that represents how much money she has left to spend on clothes.
- **28.** The solution set of which inequality is represented in the number line below?

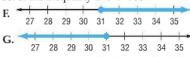
A.
$$x + 6 \le 14$$

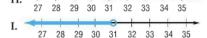
C.
$$x + 6 < 14$$

B.
$$x + 6 \ge 14$$

D.
$$x + 6 > 14$$

29. Which of the following shows the solution set of the inequality w + 4 < 35?







State whether each inequality is true or false for the given value. (Lesson 4-3A)

30.
$$18 - n > 4$$
, $n = 11$

31.
$$13 + x < 21, x = 8$$

32.
$$34 \le 5p$$
, $p = 7$

33. FOOTBALL In football, a touchdown with an extra point is worth 7 points and a field goal is worth 3 points. The winning team scored 27 points. The score consisted of two field goals, and the rest were touchdowns with extra points. Write and solve an equation to determine how many touchdowns the winning team scored. (lesson 4-2C)

State the slope and y-intercept for the graph of each equation. (Lesson 3-2A)

34.
$$y = 3x + 2$$

35.
$$y = -\frac{x}{2} - 1$$

36.
$$y = -2x + 4$$

37. MONEY The table at the right shows how much Julianne earned per paycheck. If she saved 28% of her earnings, how much money did she save? (Lesson 1-2C)

Paycheck	Total Earnings (\$)	
1	272	
2	298	
3	304	

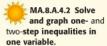


One-Step Inequalities

Main Idea

Solve and graph one-step inequalities by using the Multiplication or **Division Properties** of Inequality.

NGSSS







Solving Inequalities by Multiplication or Division

COINS Lamar, Oscar, and Nick put the money from their pockets on the table. Lamar has more money than Nick. Will this still be true if each boy spends half of his money?

Name	Money	Amount
Lamar	1 dollar bill, 2 quarters, 2 dimes	\$1.70
Oscar	1 dollar bill, 3 quarters, 1 dime, 1 nickel	\$1.90
Nick	5 quarters, 1 dime, 1 nickel	\$1.40

- 1. Divide each side of the inequality 1.70 > 1.40 by 2. Is the resulting inequality true or false?
- 2. Oscar and Lamar each tripled their money by doing lawn work. Who has more now?

The example above demonstrates additional properties of inequality.

Key Concept

Properties of Inequality

Words

When you multiply or divide each side of an inequality by a positive number, the inequality remains true.

For all numbers a, b, and c, where c > 0,

1. if a > b, then ac > bc and $\frac{a}{c} > \frac{b}{c}$.

2. if a < b, then ac < bc and $\frac{a}{c} < \frac{b}{c}$.

Examples

$$2 > -10$$
 $2 > -10$

$$\frac{2}{2} > \frac{-10}{2}$$

20 < 32

$$\frac{2}{2} > \frac{2}{2}$$

1 > -5

These properties are also true for $a \ge b$ and $a \le b$.

Study Tip

Checking Solutions You can check the solution in Example 1 by substituting numbers greater than -6into the inequality and testing it to verify that it holds true.

EXAMPLES

Solve Inequalities

Solve 7y > -42. Graph the solution set on a number line. 7y > -42 Write the inequality.

Division Property of Inequality



Draw an open dot at -6 with an arrow to the right.





2) Solve $\frac{1}{3}x \le 8$. Graph the solution set on a number line.

$$\frac{1}{3}x \le 8$$
 Write the inequality.

$$3\left(\frac{1}{3}x\right) \le 3(8)$$
 Multiplication Property of Inequality

$$x \le 24$$
 Simplify.

The solution is $x \le 24$.

Graph the solution set.





Solve each inequality. Graph the solution set on a number line.

a.
$$3a \ge 45$$

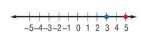
b.
$$\frac{n}{4} < -16$$
 c. $81 \le 9p$

Draw a closed dot at 24

with an arrow to the left.

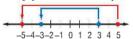
What happens when each side of an inequality is multiplied or divided by a negative number?

Graph 3 and 5 on a number line.



Since 3 is to the left of 5, 3 < 5.

| Multiply each number by -1.



Since -3 is to the right of -5, -3 > -5.

The numbers being compared switched positions as a result of being multiplied by a negative number. In other words, their order reversed.

Study Tip

Common Error Do not reverse the inequality symbol just because there is a negative sign in the inequality, as in 7y < -42. Only reverse the inequality symbol when you multiply or divide each side by a negative number.

Key Concept

Properties of Inequality

Words

When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be reversed for the inequality to remain true.

Symbols

For all numbers a, b, and c, where c < 0,

1. if
$$a > b$$
, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

2. if
$$a < b$$
, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

Examples

$$8 > 5$$
 $-3 < 9$ $-1(8) < -1(5)$ Reverse the inequality symbols. $\frac{-3}{-3} > \frac{9}{-3}$

-8 < -5

These properties are also true for $a \ge b$ and $a \le b$.



EXAMPLES Multiply or Divide by a Negative Number

Solve each inequality. Graph the solution set on a number line.



$$\frac{a}{2} \ge 8$$
 Write the inequality.

$$-2\left(\frac{a}{-2}\right) \le -2(8)$$
 Multiplication Property of Inequality; reverse inequality symbol. $a \le -16$ Simplify.

$$\bigcirc -24 > -6n$$

$$-24 > -6n$$
 Write the inequality.

$$\frac{-24}{-6} < \frac{-6n}{-6}$$
 Division Property of Inequality; reverse inequality symbol.

$$4 < n \text{ or } n > 4$$
 Simplify.



d. $\frac{c}{-7} < -14$

e.
$$-5d \ge 30$$
 f. $-3 \le \frac{v}{-8}$



Real-World Link • • • • •

More than 400,000 people participate each year in marathons. The most popular marathon is the New York Marathon.

Real-World EXAMPLE

MARATHONS Dyani is training for a marathon by running no less than 45 kilometers per week. She runs at an average rate of 12 kilometers per hour. Write and solve an inequality to find the minimum number of hours she should run. Interpret the solution.

The phrase *no less than* means *greater than or equal to*. Let h = thenumber of hours she runs in one week.

 $12h \ge 45$ Write the inequality.

$$\frac{12h}{12} \ge \frac{45}{12}$$
 Division Property of Inequality

$$h \geq 3.75$$
 Simplify.

Dyani should run for at least 3.75 hours, or 3 hours 45 minutes, per week.



CHECK Your Progress

g. GRADES Each of the 20 questions on a math test is worth 3 points. Write and solve an inequality to find how many questions you must answer correctly to earn a score of at least 45 points. Interpret the solution.



Examples 1-4 (pp. 251-253)

Solve each inequality. Graph the solution set on a number line.

1. 3x > 12 **2.** $\frac{1}{2} > \frac{1}{3}y$ **3.** $8x \le -72$ **4.** $\frac{h}{4} \ge -6$ **5.** -4y > 32 **6.** $-56 \le -7p$ **7.** $\frac{g}{-2} < -7$ **8.** $\frac{d}{-3} \ge -3$

Example 5 (p. 253)

9. MUSIC Monique wants to practice the piano at least 6 hours each week. If she averages 1.5 hours per day, write and solve an inequality to find how many days she will have to practice to have at least 6 hours of practice per week. Interpret the solution.

Practice and Problem Solving



= Step-by-Step Solutions begin on page R21. Extra Practice is on page EP14.

Examples 1-4 (pp. 251-253)

Solve each inequality. Graph the solution set on a number line.

10. 5x < 15

11. 9n ≤ 45

 13. -12 > 3g 14. $-100 \le 50p$ 15. 2y < -22

 16. $\frac{x}{9} \le -3$ 17. $\frac{n}{7} < -14$ 18. $\frac{m}{-2} < -7$

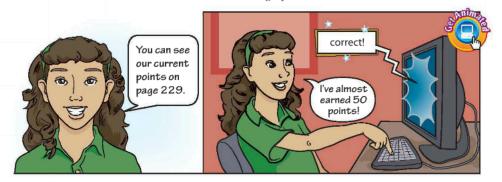
 19. $\frac{t}{-5} \le -2$ 20. $-8 \le \frac{y}{0.2}$ 21. $\frac{-1}{2}k > -10$

Example 5 (p. 253)

22. GYM MEMBERSHIP A gym charges \$5 each time you go, or you can buy a yearly membership for \$190. Write and solve an inequality to find how many times a person should use the gym so that a yearly membership is less expensive than paying each time. Interpret the solution.

WORK Max charges \$6 an hour to rake leaves. He is saving money for shoes that cost \$89. Write and solve an inequality to find how many whole hours Max must work to buy the shoes. Interpret the solution.

24. GRAPHIC NOVEL Refer to the graphic novel frame below for Exercises a–b.



- a. In order to win the game, a player must earn 50 points. How many more points does each player need to earn 50 points?
- b. Write and solve an inequality to determine how many questions each player needs to answer correctly to earn a total of 50 points.



H.O.T. Problems

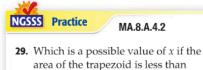
- 25. OPEN ENDED Write an inequality for the following sentence and then solve. Then name three numbers that are possible solutions. Explain. The quotient of a number and -6 increased by 5 is at most 9.
- **26. CHALLENGE** The product of an integer and -12 is less than -132. What is the least integer that meets this condition?
- 27. Which One Doesn't Belong? Identify the inequality that does not belong with the other three. Explain your reasoning.

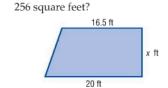
-10 > -5x

4a < -16

-b > 12

Write MATH Explain when you should reverse the inequality symbol when solving an inequality.





A. 14

C. 16

B. 15

D. 17

30. If 3n > 18, then n could be which of the following values?

F. 2

H. 6

G. 4

I. 8

31. Which of the following shows the solution set of $\frac{x}{-4} \ge -3$?

13 14

13 14

12 13 14

10 11 12 13 14 15 16 17 18 32. SHORT RESPONSE Solve the inequality $\frac{-x}{5} \le -6$.



Solve each inequality. Graph the solution set on a number line. (Lesson 4-38)

33. y + 7 < 9

34. a - 5 < 2

35. $j-9 \ge -12$ **36.** -14 > 8 + n

Write an inequality for each sentence. (Lesson 4-3A)

- 37. A minimum speed on a certain highway is 45 miles per hour.
- 38. A hummingbird's wings can beat up to 200 times per second.